Already 80 years old, it is difficult to believe. But I am lucky to be a mathematician. It is better than being a top model, or a tennis player. Physical decline is inevitable, but who cares, on s'en fiche, provided the brain still functions...

I want to thank the organizers of this conference as well as all the speakers. Their presence deeply touches me. I am also sad to have learned that Frances Kirwan could not come since her husband is in hospital, and that Jesus de Loera could not come since he himself is in hospital. I wish them the best.

Now I thought that it might be interesting for some of you to know why this conference has different themes, to know what subjects I abandoned, and the ones I pursued, alone or with wonderful collaborators. I will try to sketch some of these encounters.

So 60 years ago, that is yesterday, I wanted to try to do research in Math, and I had chosen Chevalley as a thesis director. He gave me as a subject the deformations of nilpotent Lie algebras, a subject that was of interest to no one. The first seminar which I timidly attended was organised by Chevalley with the purpose of studying the book in german of Hirzebruch : Neue topologisches Methoden in der Algebraischen Geometrie. Hirzebruch had proven the Riemann-Roch theorem for any projective varieties, the first enormous progress 100 years after the theorem that Riemann and his student Roch had proven for curves. So I gave a talk on the first chapter of the book : The function $x/sin(x)$ and its powers. My understanding of the following chapters was equal to zero. Furthermore, Chevalley was telling me that he was dépassé, outdated, that I should go to Grothendieck's seminar, to the place where things were happening. Very obediently, I would go on the RER to the IHES every week, sit alone in the corner of the seminar room, and come back alone on the RER, fearing that someone would sit next to me. In short, like my thesis director, I was depressed by schemes, commutative diagrams and Grothendieck. I thought I had better give up the idea of doing research.

Miraculously, Dixmier who was very interested in nilpotent groups had heard about me and also knew the work of Monique Nahas on deformations of the representations of the Poincaré group. So I finally found myself included in the theoretical physics group of the Ecole Polytechnique. And guided by Monique Nahas, I learned about unitary representations of Lie groups. I thought I would never hear anything anymore about algebraic geometry and Todd classes. Instead, I learned the uncertainty principle, quantum mechanics, particle physics with electrons jumping happily from an orbit to another. And, now, nice electrons are ready to help us to make computations on a quantum computer.

What a beautiful theorem, Bargmann's classification of all unitary representations of $SL(2,R)$! What a beautiful theorem, Stone-Von Neumann's theorem on the uniqueness of representations of the commutation relations $PQ-QP=1$, and as a consequence the construction of the metaplectic representation of the symplectic group, finite dimensional, infinite dimensional, over the p-adic or adeles !

The problem which everybody around me was excited about at the time was the determination of all unitary representations of all Lie groups. Why not ? these questions were progressing very rapidly, with Harish-Chandra, Dixmier, Kirillov, Kostant, Pukánszky, Duflo and many others.

With the help of Hugo Rossi, then Masaki Kashiwara, I proved some theorems on the subject of small unitary representations of reductive groups, by using Fourier transforms of measures on cones, or by using the metaplectic representation. With Gerard Lion, we improved the understanding of special cases of the Shintani correspondance. I also discovered a very beautiful equality between sums of orbital integrals and the Plancherel formula that I called the Poisson-Plancherel formula. Harish-Chandra formulae for the Plancherel measure are so beautiful that I was happy to get them in my way.

Now I regret not to have done more on these themes. Understanding the p-adic analogues, symmetric spaces and Plancherel formula, general orbital integrals, small representations and Theta correspondences, an infinite number of themes that I abandoned thinking that I would come back to these themes later, but now it is too late...

I also regret that I did not follow the development of representations of quantum groups, associated with the new development of theoretical physics, where particles are replaced by strings. But I am happy that I had very good students, who are in love with these hard core subjects. I thank them for giving talks here ;

But back to historical considerations. In 1968, Kirillov conjectured the "universal formula for characters". This formula for compact Lie groups was a consequence of Weyl's formula, but this formula is of a very different nature, in particular it had the pretention to hold true for all Lie groups and all irreducible unitary representations, finite or infinite dimensional. Going from Weyl's formula to Kirillov's universal formula is a typical example of a delocalization formula in equivariant cohomology, as we understood much later with Nicole Berline.

Then Michel Duflo proved an essential philosophical theorem : the spectrum of the center of the envelopping algebra of any Lie algebra is the space of coadjoint orbits. And guess what ? The function $x/sin(x)$ was striking back !

These discoveries deeply influenced my own research. I want to thank Michel Duflo for his constant support, while I must confess that, back then, I was envying his mathematical successes.

So let me quickly go over the rest of my mathematical life.

Around the years 1980, there were many changes in my life. First I had a baby.. who was of a great help to me since the first day of her life. Second, Witten imagined a "proof" of the Atiyah-Singer index formula via Duitermaat-Heckman's exact stationary phase. Third, Bismut transformed Witten intuition into a proof.

I started working with Nicole Berline. We met Ezra Getzler. So with the help of the harmonic oscillator, heat kernel asymptotics, equivariant cohomology, we were back working on the Riemann-Roch formulae, and equivariant index formulae, but this time it was through marvelous exact formulae, not just dull commutative diagrams.

The conjecture $[Q, R] = 0$ (quantization commutes with reduction) of Guillemin-

Sternberg on multiplicities became a painful obsession, painful, since I was not getting anywhere. But later Paul-Emile Paradan understood in a way that I liked why geometric quantization of a symplectic manifold satisfies this miraculous property of $[Q, R] = 0$, proved by Meintrenken and Sjamaar. And we developed his ideas on $[Q, R] = 0$ much further, to the case of line bundles with a curvature not necessarily non degenerate and to non compact manifolds with proper Kostant moment map.

The tools we developped with Shrawan Kumar on equivariant cohomology with generalized coefficients are essential whenever the manifold is non compact, or the operators not elliptic. With this tool, together with Nicole Berline, we wrote a formula for indices of transversally elliptic operators.

Our work with Paul-Emile Paradan and Yiannis Loizides on Semi-classical analysis of piecewise quasi-polynomial functions is a strong contribution for the understanding of the deep relation between continuous and discrete formulae, as in quantum mechanics. Duistermaat-Heckman's measure and generalizations illustrates the wave behavior of particles, while multiplicities illustrate the discrete behaviour of spectrum of particles. BUT, as is apparent in the famous Euler-MacLaurin formula, one can compute exactly discrete sums by means of continuous formulae. This is also the spirit of our articles on splines and transversally elliptic indices with De Concini and Procesi. Spline functions occuring in index multiplicities are very special and satisfy wall crossing formulae. For example, I am happy that our formulae with Arzu Boysal can be used to study $[Q, R] = 0$ for quantization of moduli spaces and Verlinde formula ; Happy and unhappy, since I think probably wrongly that we could have done it ourselves.

Perhaps, I would also like to say something about my excursion with Welleda Baldoni, Nicole Berline and Charles Cochet to the strange land of algorithms. Studying integer points in polytopes was quite natural since for toric varieties one can compute the number of integral points in the associated polytope by the Riemann-Roch formula, even for the very singular case as we proved with Michel Brion. I was invited in Italy to give a talk on this subject. So I wanted to compute a significant example. For instance, a nice polytope is the polytope of doubly stochastic $n \times n$ matrices, coefficients in rows and lines adding to 1. So I asked Velleda Baldoni to help me compute this example. Although our work with Brion and Szenes gives a very simple residue formula for it, we learned from Matthias Beck that the computation takes an infinite time (500 hours for $n = 9$, more than 1 year of computer time, for $n = 10$). So we (Baldoni, Berline and myself) started to understand that time is not infinite (and I understand this better and better); Nice formula, but how long would it take to compute it? Now I always ask myself this question that few mathematicians care about. We enjoyed working on these completely new questions for us with new colleagues. We also learned to be modest. One cannot ask for too much. For example, thanks to the saturation theorem of Knutson-Tao, one can ask if a Clebsh-Gordon coefficient is not zero and get an answer, but we cannot ask what is its value. I mean, one can ask, but we will not get the answer; or we have to be content with approximate formulae, exact formulae only being useful to test algorithms. The talks of Barvinok and of Michael Walter will be concerned with this subject.

Now what ?

I think I am now interested in representation theory without representations, as in deformation quantization and asymptotics. For example, I would like to prove a Kirillov like trace formula for equivariant deformation quantization : in particular a formula valid for non compact symplectic manifolds. My friend Ezra Getzler tells me that it is a good problem for a student, while my friend Andras Szenes tells me that at my age I should not be too ambitious. So after all it might be a good problem for me.